**Question 11** **(7 marks)**

(a) Four random variables $W$, $X$, $Y$ and $Z$ are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these.

(4 marks)

*The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.*

(i) $W$ is the number of throws of a dice until a six is scored.

(ii) $X$ is the score when a dice is thrown.

(iii) $Y$ is the number of odd numbers showing when a dice is thrown.

(iv) $Z$ is the total of the scores when two dice are thrown.

(b) Pegs produced by a manufacturer are known to be defective with probability $p$, independently of each other. The pegs are sold in bags of $n$ for $4.95. The random variable $X$ is the number of faulty pegs in a bag.

If $E(X) = 1.8$ and $Var(X) = 1.728$, determine $n$ and $p$. (3 marks)

**Question 11** **(7 marks)**

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of $n$ students is selected from all Year 12's in this country, and the random variable $X$ is the number of those in the sample who study advanced mathematics.

(a) Describe the distribution of $X$. (2 marks)

(b) If $n = 22$, determine the probability that

(i) three of the students in the sample study advanced mathematics. (1 mark)

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

(iii) none of the students in the sample study advanced mathematics. (1 mark)

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

**Question 13** **(9 marks)**

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable $X$ be the number of first grade avocados in a single tray.

(a) Explain why $X$ is a discrete random variable, and identify its probability distribution. (2 marks)

(b) Calculate the mean and standard deviation of $X$. (2 marks)

(c) Determine the probability that a randomly chosen tray contains

(i) 18 first grade avocados. (1 mark)

(ii) more than 15 but less than 20 first grade avocados. (2 marks)

(d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

**Question 14** **(14 marks)**

(a) Determine the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

(b) A Bernoulli trial, with probability of success $p$, is repeated $n$ times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine $n$ and $p$. (4 marks)

**Question 15** **(10 marks)**

A slot machine is programmed to operate at random, making various payouts after patrons pay $2 and press a start button. The random variable $X$ is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, $P$, that the machine makes a certain payout, $x$, is shown in the table below.

| Payout ($) $x$ | 0 | 1 | 2 | 5 | 10 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|---|---|
| Probability $P(X = x)$ | 0.25 | 0.45 | 0.2125 | 0.0625 | 0.0125 | 0.005 | 0.005 | 0.0025 |

(a) Determine the probability that

(i) in one play of the machine, a payout of more than $1 is made. (2 marks)

(ii) in ten plays of the machine, it makes a payout of $5 no more than once. (2 marks)

(iii) in five plays of the machine, the second payout of $1 occurs on the fifth play. (3 marks)

(b) Calculate the mean and standard deviation of $X$. (2 marks)

(c) In the long run, what percentage of the patron's money is returned to them? (1 mark)

**Question 17** **(10 marks)**

Let the random variable $X$ be the number of vowels in a random selection of four letters from those in the word LOGARITHM, with no letter to be chosen more than once.

(a) Complete the probability distribution of $X$ below. (1 mark)

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{5}{42}$ | $\frac{10}{21}$ | | $\frac{1}{21}$ |

(b) Show how the probability for $P(X = 1)$ was calculated. (2 marks)

(c) Determine $P(X \geq 1 | X \leq 2)$. (2 marks)

Let event $A$ occur when no vowels are chosen in random selection of four letters from those in the word LOGARITHM.

(d) State $P(\bar{A})$. (1 mark)

(e) Let $Y$ be a Bernoulli random variable with parameter $p = P(A)$. Determine the mean and standard deviation of $Y$. (2 marks)

(f) Determine the probability that $A$ occurs no more than twice in ten random selections of four letters from those in the word LOGARITHM. (2 marks)

**Question 11** **(7 marks)**

(a) Four random variables $W$, $X$, $Y$ and $Z$ are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these. (4 marks)

*The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.*

(i) $W$ is the number of throws of a dice until a six is scored.

| Solution |
|---|
| Neither - distribution is geometric |
| **Specific behaviours** |
| ✓ answer with reason |

(ii) $X$ is the score when a dice is thrown.

| Solution |
|---|
| Uniform - all outcomes are equally likely |
| **Specific behaviours** |
| ✓ answer with reason |

(iii) $Y$ is the number of odd numbers showing when a dice is thrown.

| Solution |
|---|
| Bernoulli - two complementary outcomes |
| **Specific behaviours** |
| ✓ answer with reason |

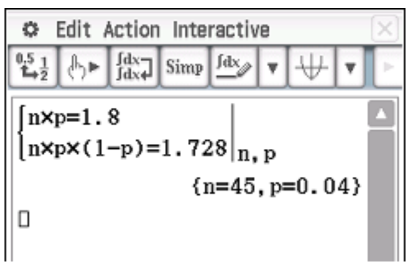
(iv) $Z$ is the total of the scores when two dice are thrown.

| Solution |
|---|
| Neither - distribution is triangular |
| **Specific behaviours** |
| ✓ answer with reason |

(b) Pegs produced by a manufacturer are known to be defective with probability $p$, independently of each other. The pegs are sold in bags of $n$ for $4.95. The random variable $X$ is the number of faulty pegs in a bag.

If $E(X) = 1.8$ and $Var(X) = 1.728$, determine $n$ and $p$. (3 marks)

| Solution |
|---|
| $np = 1.8, np(1-p) = 1.728$ <br> $\therefore 1 - p = \dfrac{1.728}{1.8} = 0.96$ <br> $p = 0.04$ <br> $n = \dfrac{1.8}{0.04} = 45$ |
| **Specific behaviours** |
| ✓ writes equations for mean and variance <br> ✓ solves for $p$ <br> ✓ solves for $n$ |



**Question 11** **(7 marks)**

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of $n$ students is selected from all Year 12's in this country, and the random variable $X$ is the number of those in the sample who study advanced mathematics.

(a) Describe the distribution of $X$. (2 marks)

| Solution |
|---|
| $X \sim B(n,\ 0.15)$ - binomial distribution with $n$ trials and $p = 0.15$. |
| **Specific behaviours** |
| ✓ states binomial distribution<br>✓ states parameters of binomial distribution |

(b) If $n = 22$, determine the probability that

(i) three of the students in the sample study advanced mathematics. (1 mark)

| Solution |
|---|
| $P(X = 3) = 0.2370$ |
| **Specific behaviours** |
| ✓ evaluates probability |

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

| Solution |
|---|
| $P(X \geq 4) = 0.4248$ |
| **Specific behaviours** |
| ✓ evaluates probability |

(iii) none of the students in the sample study advanced mathematics. (1 mark)

| Solution |
|---|
| $P(X = 0) = 0.0280$ |
| **Specific behaviours** |
| ✓ evaluates probability |

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

| Solution |
|---|
| $Y \sim B(10,\ 0.028)$<br>$P(Y \geq 1) = 0.247$ |
| **Specific behaviours** |
| ✓ states binomial distribution with parameters<br>✓ evaluates probability |

**Question 13** **(9 marks)**

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable $X$ be the number of first grade avocados in a single tray.

(a) Explain why $X$ is a discrete random variable, and identify its probability distribution. (2 marks)

| Solution |
|---|
| $X$ is a DRV as it can only take integer values from 0 to 24.<br>$X$ follows a binomial distribution: $X \sim B(24, 0.75)$ |
| **Specific behaviours** |
| ✓ explanation using discrete values<br>✓ identifies binomial, with parameters |

(b) Calculate the mean and standard deviation of $X$. (2 marks)

| Solution |
|---|
| $\bar{X} = 24 \times 0.75 = 18$<br>$\sigma_x = \sqrt{18 \times 0.25} = \frac{3\sqrt{2}}{2} \approx 2.12$ |
| **Specific behaviours** |
| ✓ mean, ✓ standard deviation |

(c) Determine the probability that a randomly chosen tray contains

(i) 18 first grade avocados. (1 mark)

| Solution |
|---|
| $P(X = 18) = 0.1853$ |
| **Specific behaviours** |
| ✓ probability |

(ii) more than 15 but less than 20 first grade avocados. (2 marks)

| Solution |
|---|
| $P(16 \le X \le 19) = 0.6320$ |
| **Specific behaviours** |
| ✓ uses correct bounds<br>✓ probability |

(d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

| Solution |
|---|
| $P(X \le 11) = 0.0021$<br>$0.0021 \times 1000 \approx 2 \text{ trays}$ |
| **Specific behaviours** |
| ✓ identifies upper bound and calculates probability<br>✓ calculates whole number of trays |

**Question 14** **(14 marks)**

(a) Determine the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

| Solution |
|---|
| $p(1-p)=0.24$<br>$p=0.4,\ 0.6 \Rightarrow$ mean is either 0.4 or 0.6 |
| **Specific behaviours** |
| ✓ writes variance equation<br>✓ solves equation<br>✓ states both values of p are possible means |

(b) A Bernoulli trial, with probability of success $p$, is repeated $n$ times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine $n$ and $p$. (4 marks)

| Solution |
|---|
| $X \sim B(n,p)$<br>$np=5.76$<br>$np(1-p)=1.92^2$<br>$1-p=1.92^2 \div 5.76=0.64$<br>$p=0.36$<br>$n=16$ |
| **Specific behaviours** |
| ✓ identifies distribution of successes as binomial<br>✓ states equation for mean<br>✓ states equation for variance (or standard deviation)<br>✓ solves equations for $n$ and $p$ |

(c) The probability that a student misses their bus to school is 0.2, and the probability that they miss the bus on any day is independent of whether they missed it on the previous day.

Over five consecutive weekdays, what is the probability that the student

(i) only misses the bus on Tuesday? (2 marks)

| Solution |
|---|
| $0.2 \times 0.8^4 = 0.08192$ |
| **Specific behaviours** |
| ✓ uses 0.8 for not catching bus<br>✓ determines probability |

(ii) misses the bus at least twice? (2 marks)

| Solution |
|---|
| $X \sim B(5, 0.2)$<br>$P(X \geq 2) = 0.26272$ |
| **Specific behaviours** |
| ✓ identifies binomial situation<br>✓ evaluates cumulative probability |

(iii) misses the bus on Tuesday and on two other days? (3 marks)

| Solution |
|---|
| $P = 0.2 \times P(Y = 2)$ where $Y \sim B(4, 0.2)$<br>$P(Y = 2) = 0.1536$<br>$P = 0.2 \times 0.1536$<br>$= 0.03072$ |
| **Specific behaviours** |
| ✓ identifies binomial situation for other two days<br>✓ evaluates probability of missing bus on two other days<br>✓ determines probability |

**Question 15** **(10 marks)**

A slot machine is programmed to operate at random, making various payouts after patrons pay \$2 and press a start button. The random variable $X$ is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, $P$, that the machine makes a certain payout, $x$, is shown in the table below.

| Payout (\$) $x$ | 0 | 1 | 2 | 5 | 10 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|---|---|
| Probability $P(X = x)$ | 0.25 | 0.45 | 0.2125 | 0.0625 | 0.0125 | 0.005 | 0.005 | 0.0025 |

(a) Determine the probability that

(i) in one play of the machine, a payout of more than \$1 is made. (2 marks)

| Solution |
|---|
| $P(X > 1) = 1 - (0.25 + 0.45) = 0.3$ |
| **Specific behaviours** |
| ✓ states required probability<br>✓ calculates probability |

(ii) in ten plays of the machine, it makes a payout of \$5 no more than once. (2 marks)

| Solution |
|---|
| $Y \sim B(10, 0.0625)$<br>$P(Y \leq 1) = 0.8741$ |
| **Specific behaviours** |
| ✓ indicates binomial distribution<br>✓ calculates probability |

(iii) in five plays of the machine, the second payout of \$1 occurs on the fifth play. (3 marks)

| Solution |
|---|
| First payout in one of four plays:<br>$W \sim B(4, 0.45)$<br>$P(W = 1) = 0.2995$<br>Second payout:<br>$P = 0.2995 \times 0.45 = 0.1348$ |
| **Specific behaviours** |
| ✓ uses first and second event<br>✓ calculates $P$ for first event<br>✓ calculates $P$ for both events |

**Question 17** **(10 marks)**

Let the random variable $X$ be the number of vowels in a random selection of four letters from those in the word LOGARITHM, with no letter to be chosen more than once.

(a) Complete the probability distribution of $X$ below. (1 mark)

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{5}{42}$ | $\frac{10}{21}$ | $\mathbf{\frac{5}{14}}$ | $\frac{1}{21}$ |

| Solution |
|---|
| $1 - \left(\frac{5}{42} + \frac{10}{21} + \frac{1}{21}\right) = \frac{5}{14}$ |
| **Specific behaviours** |
| ✓ uses sum of probabilities |

(b) Show how the probability for $P(X = 1)$ was calculated. (2 marks)

| Solution |
|---|
| $P(X = 1) = \frac{\binom{3}{1} \times \binom{6}{3}}{\binom{9}{4}} = \frac{3 \times 20}{126} = \frac{10}{21}$ |
| **Specific behaviours** |
| ✓ uses combinations for numerator<br>✓ uses combinations for denominator and simplifies |

(c) Determine $P(X \geq 1 | X \leq 2)$. (2 marks)

| Solution |
|---|
| $P = \frac{\frac{10}{21} + \frac{5}{14}}{\frac{20}{21}} = \frac{5/6}{20/21} = \frac{7}{8}$ |
| **Specific behaviours** |
| ✓ obtains numerator<br>✓ obtains denominator and simplifies |

Let event $A$ occur when no vowels are chosen in random selection of four letters from those in the word LOGARITHM.

(d) State $P(\bar{A})$. (1 mark)

| Solution |
|---|
| $P(\bar{A}) = 1 - \frac{5}{42} = \frac{37}{42}$ |
| **Specific behaviours** |
| ✓ calculates probability |